ALTAM Spring 2026 Model Solutions and Examiners' Comments

Question 1 See Excel

Question 2

Examiners' Comments:
Overall, this question was done well by most candidates. The numerical parts scored significantly higher than the verbal parts.

(a)

(i) $\Pr_0 = -100$

(ii) $\Pr_1 = \left(0.95G - 50\right)(1.03) - \left(1 - p_{xy}\right)\left(10{,}000\right)$

$$-0.1 p_{xy}\left(0.5G\right) = 400$$

$p_{xy} = 0.98 \times 0.96 = 0.9408$

$$\Rightarrow G = \frac{400 + 50(1.03) + 0.0592(10{,}000)}{0.95(1.03) - (0.1)\left(0.5\right)(0.9408)} = 1{,}120.28$$

(iii) $\Pr_2 = \left(0.95G - 50\right)(1.03) - \left(1 - p_{x+1:y+1}\right)\left(10{,}000\right) - p_{x+1:y+1}\left(500\right)$

$p_{x+1:y+1} = 0.97 \times 0.95 = 0.9215$

$\Rightarrow \Pr_2 = -201.05$

Examiners' Comments:

1. *Candidates generally did very well on this part.*

(b) (i) $\Pi = \left(-100,\ 400,\ -201.05(0.9\,p_{xy})\right) = \left(-100,\ 400,\ -170.24\right)$

(ii) $NPV = -100 + 400 v_{10\%} - 170.24 v_{10\%}^2 = 122.95$

(iii) $pm = \dfrac{NPV}{G\left(1 + 0.9\,p_{xy} v_{10\%}\right)} = \dfrac{122.95}{1982.62} = 6.20\%$

Examiners' Comments:

1. *Candidates generally did very well on this part.*
2. *The most common errors were omitting the 0.9 term in the survival probability, and using the earned interest rate instead of the hurdle rate to calculate the NPV and profit margin.*

(c) (i) A negative projected cash flow implies a need for new money. It is imprudent to create a product that requires funding mid-term, without making provision for that funding (i.e. setting reserves).

(ii) $\left({}_1V + 0.95G - 50\right)\left(1.03\right) - 10000(1 - p_{x+1:y+1}) - 500\,p_{x+1:y+1} = 0$

$\Rightarrow {}_1V = \left(10000(0.0785) + 500(0.9215)\right)/1.03 + 50 - .95G$

$= 195.20$

Or: $\Pr_2^{new} = \Pr_2^{old} + {}_1V(1.03) = -201.05 + {}_1V(1.03) = 0$

$$\Rightarrow {}_1V = \frac{201.05}{1.03} = 195.20$$

(iii) $\Pr_1^{new} = Pr_1^{old} - \left(0.9\,p_{xy}\right)\left({}_1V\right) = 234.73$

$\Rightarrow \Pi^{new} = \left(-100,\, 234.73,\, 0\right)$

$\Rightarrow NPV = -100 + 234.73v_{10\%} = 113.39$

(iv) The hurdle rate is 10%; the reserves are only earning 3%. Higher reserves implies pulling funds out of the pool that is potentially earning 10% and investing them at only 3%, which leads to less profit for the insurer.

Examiners' Comments

1. *Parts (ii) and (iii) were done fairly well. Parts (i) and (iv), which tested candidates' understanding of the fundamental principles of reserving and profit testing, were done less well.*
2. *For full credit in part (iv), candidates were required to note the difference between the earned rate and the hurdle rate.*

(d) (i) Examples of factors to take into consideration

- The health status of (x) and (y) at that time; less likely to surrender if they believe their mortality risk is high.
- The ability of the policyholders to afford the second year premium (perhaps their circumstances have changed).
- The cost of purchasing insurance cover through a new 1-year policy if they do surrender, noting that the cost would be offset by the 50% return of premium benefit from the original policy.
- Changing needs for insurance cover -- perhaps their circumstances have changed such that a different product would be more suitable.

(ii) Since we expect the surrendering policyholders to be healthier than average, the remaining policyholders would be expected to have higher mortality than the surrendering policyholders, through adverse selection.

Examiners' Comments

1. *In part (i), some candidates merely listed factors. The key word in the question was "describe", which signals that the examiners are looking for (a little) more than a list of words. For full credit, candidates were required to give a brief explanation of why the factor listed is relevant.*
2. *In part (ii), some candidates gave answers that were too brief for full credit – e.g. "adverse selection means higher mortality". The word "Explain" in the question*

signals that for full credit the examiners were looking for a brief explanation as to why the non-surrendering policyholders would be expected to have higher mortality.

3. *Many candidates answered part (ii) by explaining what adverse selection is, without addressing the specific product in the question (for example, some candidates used annuities to describe adverse selection). Note that in the solution above, adverse selection is not defined, as the question did not specifically ask for a definition. Generic answers to both (i) and (ii) earned little or no credit.*

Question 3

(a) (i) $_{20}p_{50}^{01} = \left({}_{10}p_{50}^{00}\right)\left({}_{10}p_{60}^{01}\right) + \left({}_{10}p_{50}^{01}\right)\left({}_{10}p_{60}^{11}\right)$

$$= 0.16442 + 0.04690 = 0.21131$$

(ii) $\ddot{a}_{50:\overline{20|}}^{00} = \ddot{a}_{50}^{00} - v^{20}\,{}_{20}p_{50}^{00}\,\ddot{a}_{70}^{00} - v^{20}\,{}_{20}p_{50}^{01}\,\ddot{a}_{70}^{10}$

By W2term: $\ddot{a}_x^{00} = \bar{a}_x^{00} + 0.5;\quad \ddot{a}_x^{01} = \bar{a}_x^{01}$

$$\Rightarrow \ddot{a}_{50:\overline{20|}}^{00} = (11.7454 + 0.5) - v^{20}(0.49695)(4.9609 + 0.5) - v^{20}(0.21132)(0.0134)$$

$$= 11.2215$$

Examiners' Comments:
1. *Part (i) was done well by most candidates.*
2. *Candidates who attempted to calculate the probability in (i) by integration using constant transition rates earned no credit.*
3. *Part (ii) was not done as well as the Examiners had expected. In particular, most candidates omitted the third term in the expression (which accounts for the possibility of moving into State 1 and then back to State 0).*
4. *Many candidates seemed unfamiliar with applying the Woolhouse 2-term formula, which is given in the formula sheet. The Examiners strongly encourage all candidates to be as familiar as possible with all the information in the formula sheet.*

(a) The premium equation is

$$E[L] = 100{,}000\bar{A}_{50}^{02} - 0.92\,G\,\ddot{a}_{50:\overline{20|}}^{00} + 0.37G = -100$$

$$\Rightarrow G = \frac{100{,}000\bar{A}_{50}^{02} + 100}{0.92\ddot{a}_{50:\overline{20|}}^{00} - 0.37} = 3337.98$$

Examiners Comments:
1. *This part was done well by most candidates, with a majority earning full credit.*

(b) (i) $_{10}V^{(0)} = 100{,}000\bar{A}_{60}^{02} - 0.92G\,\ddot{a}_{60:\overline{10|}}^{00} = 25{,}023.79$

(ii) There is a jump in $_tV^{(0)}$ when a premium is paid, from $_tV^{(0)}$ to

$_{t^+}V^{(0)} = {}_tV^{(0)} + 0.92G$, so $_{10^+}V^{(0)} = {}_{10}V^{(0)} + 0.92G = 28{,}094.75$

(iii) There are no cash flows at time 10 in State 1, because of the premium waiver, so $_{10^+}V^{(1)} = {}_{10}V^{(1)} = 49{,}334$

Examiners Comments:
1. *Part (i) was done well by most candidates who attempted it.*
2. *Many candidates omitted (c)(ii) and (c)(iii).*

3. *The most common error amongst those who attempted it was to add the premium back onto the reserve in State 1 (for part (iii)) , similarly to part (ii).*

(c) (i) For non-integer t, we have

$$\frac{d}{dt}\,{}_tV^{(0)} = \delta\,{}_tV^{(0)} - \mu^{01}_{x+t}\left({}_tV^{(1)} - {}_tV^{(0)}\right) - \mu^{02}_{x+t}\left(S - {}_tV^{(0)}\right)$$

$$\Rightarrow \lim_{t\to 10^+}\frac{d}{dt}\,{}_tV^{(0)} = \delta\,{}_{10^+}V^{(0)} - \mu^{01}_{60}\left({}_{10}V^{(1)} - {}_{10^+}V^{(0)}\right) - \mu^{02}_{60}\left(S - {}_{10^+}V^{(0)}\right)$$

$$= (0.04879)(28{,}094.74) - 0.014086(49{,}334 - 28{,}094.74)$$
$$- 0.014518(100{,}000 - 28{,}094.74)$$
$$= 27.6498$$

(ii) ${}_{10.2}V^{(0)} \approx {}_{10+}V^{(0)} + 0.2(27.6498) = 28100.3$

Examiners' Comments:

1. *The key idea for the second half of Question 3 is that cash flows in State 0 are continuous at all times except at premium dates. So when we apply the Euler method to move forward from time 10 to 10.2, we need to start after the jump at time 10. The limit notation in the question signals that the limit is approaching time 10 from above.*
2. *A common error was to include the premium in the differential equations. This is incorrect, in this case, because the premium is discretely paid, not continuously paid..*
3. *Candidates who wrote down a generic Thiele's formula, without any application to the contract in the question received no credit.*
4. *Many candidates omitted this part.*

Question 4

(a) (i) $FAS_{64.5} = S_{23}\left(\dfrac{0.5(1.02)^{41}+(1.02)^{40}+(1.02)^{39}+0.5(1.02)^{38}}{3}\right)=174,934$

(ii) $FAS_{65} = 80,000\left(\dfrac{(1.02)^{39}+\left(1.02\right)^{40}+\left(1.02\right)^{41}}{3}\right)=176,666$

(iii) There is no AL as Jeff is a new entrant.
The normal cost is the EPV at 1/1/2026 of the benefits that will accrue in 2026, i.e.

$$NC=\frac{\alpha}{l_{23}}\left(FAS_{64.5}\times r_{64}\times v^{41.5}\times \ddot{a}^{(12)}_{64.5}+FAS_{65}\times r_{65}\times v^{42}\times \ddot{a}^{(12)}_{65}\right)$$

$$\Rightarrow \text{NC}=\frac{0.016}{73,804}\left(174,934\times 865\times v^{41.5}\times 13.882+176,666\times 8203\times v^{42}\times 13.723\right)$$

$$=752.08$$

$$\Rightarrow \text{ NC rate is } \frac{752.08}{80,000}=0.940\%$$

Examiners' Comments:

1. *Parts (i) and (ii) were done well.*
2. *Only a few candidates earned full credit for part (iii). Many thought that the NC should be 0, based on the shortcut that in some cases, NC=AL/n. In this case, AL=0 and n=0 (no past service) and so that approach will not work. This meant that candidates had to rely on first principles, i.e., using the fact that the NC pays for the EPV of the increase in the actuarial liability accruing over the next year.*
3. *Candidates who did understand the principle behind the NC calculation generally scored reasonably well.*
4. *Some lost fractional credit for not converting the NC into a rate (by dividing by salary). The Examiners again emphasize the value for candidates in re-reading the question as you work through it to avoid losing points unnecessarily.*
5. *One advantage of calculating the NC rate is as a reasonableness check. The rate should be low at the start of an employee's career (as we see in part (iii) above), and will rise as the employee nears retirement.*

(b) The projected RR is $RR_{65}=\dfrac{\left(FAS_{65}\right)\times 0.016\times 42}{S_{64}}$

where $S_{64}=80,000(1.02)^{41}=180,176$

$$\Rightarrow RR_{65}=\frac{118,720}{180,176}=0.6589$$

Examiners' Comments:

This part was done well by candidates who attempted it.

(c) The accumulated DC account value at age 65 is projected to be

$$0.01S_{23}\left((1.08)^{41}+(1.02)(1.08)^{40}+(1.02)^2(1.08)^{39}+\cdots+(1.02)^{40}(1.08)\right)$$

$$=800(1.08)^{41}\left(1+\left(\frac{1.02}{1.08}\right)+\left(\frac{1.02}{1.08}\right)^2+\cdots+\left(\frac{1.02}{1.08}\right)^{40}\right)$$

$$=800(1.08)^{41}\left(\frac{1-\left(\frac{1.02}{1.08}\right)^{41}}{1-\left(\frac{1.02}{1.08}\right)}\right)$$

$$=305,428$$

This converts to an additional annual pension of

$$\frac{305,428}{\ddot{a}_{65}^{(12)}}=\frac{305,428}{13.723}=22,257,$$

which gives a total projected replacement rate of

$$\frac{118,720+22,257}{180,176}=78.24\%$$

Examiners' Comments:

1. *This part was done well by most candidates.*
2. *Many candidates did the calculation in Excel. For full credit, candidates should explain that they have used Excel, and briefly describe how they did the calculation. This allows the examiners to give partial credit in the case of minor errors.*
3. *Full credit was also given to candidates who interpreted the question as including 42 years of contributions, rather than 41.*

(d) Advantages (one required):

- There is upside opportunity from DC top up investments – potential for higher income than the DB guarantee.
- DB plans have low value for early leavers. The DC top-up will likely provide a better benefit for early leavers.
- There is contribution flexibility in the DC top up - the plan member can skip, reduce, or increase contributions.
- If the DB plan defaults or closes early, the DC top-up funds should be secure.

Disadvantages (one required):

- The DB benefit is guaranteed (as long as the plan does not default). The DC investments could do badly. A higher guaranteed benefit provides better retirement security.

Examiners' Comments:

1. *Many candidates answered this part well.*

2. *Some candidates failed to take the employee's perspective, as specified in the question, and others offered generic comments on the comparison between DB and DC plans, which didn't fully address the specific question, i.e. a comparison of a hybrid plan vs DB with higher accrual rate.*

Question 5

(a) $1000\, q_{\overline{51:61}} = 1000\left(q_{51}\ q_{61}\right) = 0.00505$

Examiners' Comments
This part was done well by almost all candidates who attempted it.

(b) (i) We have

$$AV_1 = \left(0.6P - 200 - q_{\overline{51:61}} v_{3\%}\left(FA - AV_1\right)\right)(1.05)$$

$$= \left(14,400(0.6) - 200 - \left(\frac{0.00505}{1000} v_{3\%}\right)\left(1,000,000 - AV_1\right)\right)(1.05)$$

$$= \frac{8,856.86}{1 - \left(0.00505 v_{3\%} / 1000\right)(1.05)} \qquad = 8,856.90$$

(ii) The first-year cash flows are not affected by the death of either X or Y (as long as they don't both die). Hence the AV at $t = 1$ would be the same.

Examiners' Comments:

1. *Part (i) was done well by most candidates who attempted it.*
2. *Part (ii) was done less well. Many candidates claimed that the AV at t =1 would change because the COI would change, but that would only affect future AV's not the AV at t=1.*

(c) (i) If both lives are alive at $t = 6$ then the discounted COI rate is $q_{\overline{57:67}} v_{3\%} = \left(q_{57}\right)\left(q_{67}\right) v_{3\%} = 1.7690 \times 10^{-5}$. In this case, we have

$$AV_7 = \left(AV_6 + 0.9P - 50 - q_{\overline{57:67}} v_{3\%}\left(FA - AV_7\right)\right)(1.05)$$

$$= \left(92,000 + 14,400(0.9) - 50 - \left(1.769 \times 10^{-5}\right)\left(1,000,000 - AV_7\right)\right)(1.05)$$

$$= \frac{110,136.9}{1 - \left(1.769 \times 10^{-5}\right)(1.05)} = 110,139.0$$

(ii) If X survives to $t = 6$ and Y does not, then we have

$$AV_7 = \left(AV_6 + 0.9P - 50 - q_{57} v_{3\%}\left(FA - AV_7\right)\right)(1.05)$$

$$q_{57} v_{3\%} = 0.002388$$

$$\Rightarrow AV_7 = \frac{107,648.6}{1 - \left(0.002388\right)(1.05)} = 107,919.1$$

Examiners' Comments

1. *The Examiners expected this part to be easy, and many candidates achieved full credit, but a surprising number did not.*

2. *Many candidates lost credit by trying to plug numbers into a memorized formula, rather than setting out the beginning year cash flows and accumulating them, from first principles, as shown in the solution.*
3. *A number of candidates treated the Face Amount as the Additional Death Benefit, i.e. did not deduct the year-end AV when calculating the insurance cost.*
4. *A few candidates showed no formulas or calculations, but presented an answer apparently from Excel work. Candidates are required to show their work. If Excel is used, for full credit, candidates must at least briefly explain how Excel was used. In this case, we would expect candidates using Excel to write down their formula and inputs, as well as the final answer.*

(d) (i)

$$A^{\ 1}_{\overline{61:71}:\overline{10|}} = A^{\ 1}_{61:\overline{10|}} + A^{\ 1}_{71:\overline{10|}} - A^{\ 1}_{\widehat{61:71}:\overline{10|}}$$

Using $A^{\ 1}_{x:\overline{n|}} = A_{x:\overline{n|}} - {}_nE_x$ we have

$$A^{\ 1}_{61:\overline{10|}} = 0.62201 - 0.57457 = 0.04744$$

$$A^{\ 1}_{71:\overline{10|}} = 0.63828 - 0.49848 = 0.13980$$

$$A^{\ 1}_{\widehat{61:71}:\overline{10|}} = \left(1 - d\ \ddot{a}_{61:71:\overline{10|}}\right) - (1.05)^{10}\left({}_{10}E_{61}\ {}_{10}E_{71}\right)$$

$$= 0.64554 - 0.46654 = 0.17900$$

$$\Rightarrow A^{\ 1}_{\overline{61:71}:\overline{10|}} = 0.04744 + 0.13890 - 0.17900 = 0.008236$$

OR

$$A^{\ 1}_{\overline{61:71}:\overline{10|}} = A_{\overline{61:71}} - \left({}_{10}p_{61}\right)\left({}_{10}p_{71}\right)v^{10}A_{\overline{71:81}} - \left({}_{10}p_{61}\right)\left({}_{10}q_{71}\right)v^{10}A_{71}$$

$$-\left({}_{10}q_{61}\right)\left({}_{10}p_{71}\right)v^{10}A_{81}$$

$$= 0.008236$$

(ii) If both lives are alive at t = 10, then the reserve is

$$\max\left(1{,}000{,}000A^{\ 1}_{\overline{61:71}:\overline{10|}} - AV_{10},\ 0\right)$$

$$\Rightarrow {}_{10}V = \max\left(\left(8236 - 127{,}000\right),\ 0\right) = 0$$

(iii) If only Y is alive at t = 10, then the reserve is

$${}_{10}V = \max\left(1{,}000{,}000A^{\ 1}_{71:\overline{10|}} - AV_{10},\ 0\right)$$

$$= \max\left(\left(139{,}800.5 - 127{,}000\right),\ 0\right)$$

$$=12{,}800.5$$

Examiners' Comments:

1. *Candidates generally found this to be the most challenging part of the question. Many candidates omitted this part completely.*
2. *A fairly small number of candidates correctly calculated the last survivor insurance factor in part (i).*
3. *In parts (ii) and (iii) many candidates did not deduct the AV. The AV is available to pay for the guaranteed insurance until it is fully depleted. The reserve for the secondary guarantee is the amount required to supplement the AV to ensure that the guaranteed term insurance is fully funded.*

Question 6

(a) From the formula sheet: $p(0) = kPe^{-10r}\Phi\left(-d_2(0)\right) - F_0(1-m)^{10}\Phi\left(-d_1(0)\right)$. In this case, the inputs to this formula are:

$F_0 = \left(P - 500\right), \quad kP = P = 100{,}000, \; m = 0.02$, $r = 0.04, \; \sigma = 0.25, n = 10$

$$d_1(0) = \frac{\ln\left(F_0(1-m)^{10} / P\right) + \left(r + \sigma^2 / 2\right)(10)}{\sigma\sqrt{10}} = 0.63936$$

$$\Rightarrow \Phi\left(-d_1(0)\right) = 0.26129$$

$$d_2(0) = d_1(0) - \sigma\sqrt{10} = -0.15121 \quad \Rightarrow \Phi\left(-d_2(0)\right) = 0.56009$$

$$\Rightarrow \; p(0) = 37{,}544.2 - 121{,}242.8 = 16{,}301.4$$

$${}_{10}p_{70} = \frac{l_{80}}{l_{70}} = 0.83064$$

$$\pi(0) = {}_{10}p_{70} \; p(0) = 13{,}540.6$$

ALTERNATIVE APPROACH

Work with the price of a put option on a unit of underlying stock, denoted $\tilde{p}(0)$.

Then

Let $\xi P = (P-500)(1-m)^{10} \Rightarrow \xi = 0.812987$

$$p(0) = P\xi\,\tilde{p}(0) \quad \text{where } \tilde{p}(0) = \frac{1}{\xi}e^{-10r}\Phi\left(-d_2(0)\right) - \Phi\left(-d_1(0)\right)$$

$$d_1(0) = \frac{\ln(\xi) + (r + \sigma^2 / 2)(10)}{\sigma\sqrt{(10)}} = 0.63936; \quad \Phi\left(-d_1(0)\right) = 0.26129$$

$$d_2 = d_1 - \sigma\sqrt{10} = -0.15121; \quad \Phi\left(-d_2(0)\right) = 0.56009$$

$$\Rightarrow \; \tilde{p}(0) = 0.20051$$

$$\Rightarrow \; p(0) = 100{,}000(0.812987)\,\tilde{p}(0) = 16301.37$$

$$\Rightarrow \pi(0) = {}_{10}p_{70}\,p(0) = 13540.65$$

Examiners' Comments

1. *Questions involving option valuation are usually the poorest scoring questions on ALTAM exams, and this was not an exception.*
2. *Nevertheless, part (i) was done reasonably well by most of the candidates who attempted it.*
3. *The most common error was ignoring, or partially ignoring the initial 500 expense. Candidates were awarded partial credit if the calculation was otherwise correct.*

(b) (i) The investment in the underlying assets is the short part of the delta hedge, with a market value of

$$\left(-F_0(1-m)^{10}\Phi\left(-d_1(0)\right)\right){}_{10}p_{70} = -21{,}242.8 \times 0.830645 = -17{,}645$$

(ii) A short position in stocks means that the hedge portfolio gains value when the underlying risky asset value falls. Falling underlying asset values mean increasing GMMB value, so the short position in the hedge portfolio allows it to track the GMMB value.

Examiners' Comments:

1. *Many candidates lost partial credit in (i) by not including the minus sign (it is important that the stock position is a short position) and/or by not including the survival probability in the calculation.*
2. *Few candidates earned full credit for (ii). The examiners were looking for candidates to show that they understand the underlying principle of the hedging approach, i.e. replicating the cost of the guarantee. Shorting the stock means that the hedge portfolio value increases as the stock decreases, and hence as the guarantee cost increases.*
3. *Answers that simply said, for example, that the position was short because the sign was negative earned no credit.*

(c) First assume the life survives to maturity. Let S_t denote the value of the underlying assets at t. We can assume that $S_0 = 1$ (if it does not, we can just scale S_t appropriately).

$$\Pr\left[F_{10} \le P\right] = \Pr\left[F_0 (0.98)^{10} \frac{S_{10}}{S_0} \le P\right] = \Pr\left[S_{10} \le \frac{100,000}{81,299}\right]$$

$$= \Pr\left[S_{10} \le 1.2300\right]$$

We are given that S_t follows a lognormal process, with parameters $\mu = 0.05$ and $\sigma = 0.25$. That means that for a fixed t, S_t has a lognnormal distribution with parameters $t\mu$ and $\sqrt{t}\sigma$,

$$\Rightarrow \; S_{10} \sim \text{logN}\left(10\mu, \sqrt{10}\sigma\right) \text{ i.e. } S_{10} \sim \text{logN}\left(0.5, 0.790569\right)$$

$$\Rightarrow \Pr\left[F_{10} \le P\right] = \Pr\left[S_{10} \le 1.2300\right] = \Phi\left(\frac{\ln(1.2300) - 0.5}{0.790569}\right) = 0.35548$$

The joint probability that the life survives to age 80 and that the guarantee is in the money at that time is then

$${}_{10}p_{70}(0.35548) = 0.29528$$

Examiners' Comments

1. *This part, relates to material from FAM, where it is explained that underlying the Black Scholes formula is an assumption that the risky asset price follows a lognormal process.*
2. *A small number of stronger candidates correctly identified the random variable (S_{10}) and also correctly identified the threshold for triggering the guarantee, but almost none were able to calculate the probability.*
3. *The key misunderstanding was how to convert the information about the lognnormal process into information about the probability distribution of S_{10}*

(d) (i) The minimum projected BB at time 10 is

$$BB_{10} = P\left(1.04^{10}\right) = 148,024$$

$$\Rightarrow \; \text{GMIB} = 0.105(148,024) = 15,543 \text{ p.y.}$$

(ii) The annuitization rate at age 80 is 0.105, that is

$$0.105 = \frac{1}{\ddot{a}_{80_j}} \Rightarrow \ddot{a}_{80_j} = 9.524.$$

From the tables, at $i = 0.05$, $\ddot{a}_{80_i} = 8.5484.$ As the annuity value is a decreasing function of the interest rate, it follows that $j < 0.05$

Examiners' Comments:

1. *Part (i) was done well by those that attempted it.*
2. *Part (ii) was also done fairly well.*

(e) The cost to the insurer of the GMIB is $\left(BB(0.105)\right)\ddot{a}_{80\,i}$, where *i* is the market rate of interest at maturity. This will be less than *BB* if interest rates are sufficiently high such that

$$\ddot{a}_{80\,i} < \frac{1}{0.105} = 9.5238\,.$$

Examiners Comments:

1. *This part was omitted by many candidates.*
2. *Examiners were looking for an understanding that the cost to the insurer of the GMIB depends on market rates at maturity, which will generally differ from the rate used in the guaranteed annuitization rate. The insurer pays less than the BB if interest rates are higher than j at maturity.*